Journal of Modern Technology and Engineering
Vol.5, No.1, 2020, pp.25-35

AN APPROACH TO SOLUTION OF VERBAL STATED MATHEMATICAL PROBLEMS

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Abstract. The mathematical basis of Artificial Intelligence technologies, which is one of the most popular technologies of recent years, is largely based on soft computing technologies, fuzzy logic and fuzzy sets theory. Fuzzy logic and fuzzy sets theory can be applied successfully in many areas where classical mathematics has difficulty in modeling. These fields are generally areas that contain verbal information and logical inferences other than precise numerical information. In this study, the concept of Verbal Stated Mathematical Problem (VSMP) is defined first and algorithmic solution methodology based on fuzzy logic and fuzzy sets theory of this problem is proposed. Sugeno type Fuzzy Inference System was used to construct the model of the problem. The steps of the solution algorithm are explained through the example VSMP.

Keywords: Fuzzy logic, linguistic variable, verbal stated mathematical problem (VSMP), fuzzy inference system.
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Received: 5 September 2019; Accepted: 15 February 2020; Published: 30 April 2020.

1 Introduction

Mathematics has a special place in thousands of years of human history. Whether in the construction of the Egyptian pyramids about five thousand years ago or on the basis of today's technology, mathematics and mathematical modeling can be seen. Naturally, there are certain turning points in the development of mathematical modeling. These milestones can be more strikingly exemplified by the natural development of numbers. While primitive people calculate only by using natural numbers, the number 0 (zero) is used to eliminate the congestion experienced in the solution of certain problems in the seventh century. Subsequent commercial trade et al. the need resulting from the relationships caused the negative numbers to be activated. As the complexity of the problems that mathematics attempts to solve increases, concepts such as rational numbers, irrational numbers and complex numbers have come to life.

Another stage of the adventure of mathematical modeling was the fuzzy numbers laid in 1965 by Professor Lutfi Zadeh. L. Zadeh introduced the concepts of Fuzzy Set and Fuzzy Logic to science in his article "Fuzzy Sets" published in 1965 (Zadeh, 1965, 1975). The fact that the mathematical concepts proposed in this article provide a great convenience in modeling and solving real-life problems has led to the popularization of Fuzzy Sets theory. So far, the very rigid assumptions on which mathematics relied were obstacles in modeling and solving most real-world problems, particularly social nature problems. But the fuzzy numbers, fuzzy relations, fuzzy inferences, etc. concepts that the fuzzy sets theory gives to science, paved the way for the rapid application of mathematical modeling to social natural problems (Arfi, 2010; Abdullah et al., 2004; Nasibov & Kinay, 2009).

The more important reason that fuzzy logic and fuzzy sets theory gained so much power in recent years is based on the inevitable role of this theory in technology development. In recent years, developing and expanding smart technologies, unmanned air, land, defense industry vehicles, artificial intelligence robots, etc. is based on mathematical modeling and analysis techniques such as fuzzy logic and soft computing (Melin et al., 2018; Jang et al., 1997). Other interesting approaches between problem modeling and solution technologies with fuzzy logic are granular computing approaches. In these approaches, data are converted into granulated fuzzy sets (terms) and calculations are performed on them (Pedrycz, 2013).

In spite of all the studies mentioned above, sometimes questions such as "why fuzzy logic?" and "can't these problems be solved with ordinary mathematics?" are asked. In this study, an approach that reflects the advantage of fuzzy modeling, which is perhaps the most important return of fuzzy mathematics, is discussed. The concept of Verbal Stated Mathematical Problem (VSMP), which is difficult to define in classical mathematics, was first described and a methodological approach based on fuzzy logic and fuzzy sets theory was proposed for its modeling and analysis. Through an example problem, the steps for how to define and resolve the VSMP are explained.

One of the most important operations to be done in linguistic defined problems is to know the variables, values and logical inference structures in the text. It is also possible to use advanced artificial intelligence technologies such as Text Summarization, Named Entity Recognition and Deep Learning to automate this work (Lample et al., 2016; Nadeau & Sekine, 2007; Bikel et al., 1999; Nothman et al., 2013; Turney, 2000; Yatsko et al., 2010). However, within the scope of this study, the recognition of variables, values and logical inference structures in the text, based solely on expert opinions, is discussed.

In the rest of the article, firstly the concepts of fuzzy logic and fuzzy sets theory which can be used to define and solve the Verbal Stated Mathematical Problem are given in Chapter 2. In the next, the statement of the linguistic defined problem is given in Chapter 3 and the steps required for the solution of this problem are indicated in the form of a pseudocode. In Chapter 4, modeling and solving steps of a linguistic defined problem is explained in detail through an example. In the conclusion section, the contribution of fuzzy modeling and analysis approach presented in this study is highlighted and a perspective for further studies is given.

2 Background based on fuzzy logic and fuzzy sets theory

In this sections, brief preliminaries are given about the used concepts of the fuzzy logic and fuzzy sets theory to formulate and solve the verbal stated mathematical problem.

2.1 Linguistic variables and operations on them

Formally, a linguistic (verbal) variable is defined as a quintet $(x, T(x), X, G, M)$ (Zadeh, 1975; Jang et al., 1997). Here x is the name of the variable, $T(x)$ is the set of terms (linguistic values), X is the universal set on which the terms are defined, G is syntaxis rules for construction new terms from basic terms, M is semantics of terms (membership functions that determine fuzzy values corresponding to terms). For example, if x is an "Age" variable, then the variable name will be x. The term $T(x)$ can be defined as: {young, not young, very young, not very young, middle-aged, etc.}. $X = [0, 100]$. G will be linguistic rules set for the formation of $T(x)$ terms. M will be the set of membership functions of each term in the set of terms $T(x)$.

Triangular, trapezoid, bell shaped etc. fuzzy numbers are often used to express linguistic variables. Examples of a linguistic variable "Age" and their values (terms) are given in Fig. 1.

Basic linguistic values can be combined through logic operators such as "and", "or", "not" to create linguistic values corresponding to deeper logical expressions. Moreover, the effects of linguistic values can be strengthened or weakened via using hedges such as "very", "a little bit", "approximately", "enough", "extremely" etc..

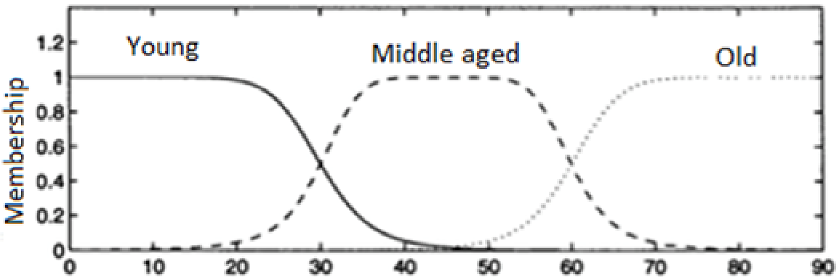


Figure 1: Example of a linguistic variable "Age" and its values

Suppose that the value of any term is expressed by the fuzzy number A, whose membership function is $A(x)$:

$$A = \int_X A(x)/x.$$

In this case, the following logical operators are often used on any of the terms A and B:

$$''\text{A and B}'' \equiv \ A(x) \wedge B(x) = \int_X \min\{A\,(x)\,, B\,(x)\}/x$$

$$''\text{A or B}'' \equiv \ \mathrm{A\,(x)} \vee \mathrm{B(x)} = \int_{\mathrm{X}} \max\{\mathrm{A\,(x)\,, B\,(x)}\}/\mathrm{x}$$

$$''\text{not A}'' \equiv \ A\,(x) = \int_X [1 - A(x)]/x.$$

Here the denotations "$\wedge$" and "$\vee$" are equivalent to the "*min*" and "*max*" operators, respectively.

Let's look at how strengthen or softening effects are defined on linguistic values. First, let's define the fuzzy number A^k as follows:

$$A^k \equiv \int_X [A(x)]^k/x.$$

In this case, the fuzzy highlighting (strengthening) operation corresponding to the envelope "very", "quite" can be given as $CON(A)$ operator defined as follows:

$$CON\,(A) \equiv A^2 = \int_X [A(x)]^2/x.$$

The fuzzy softening process, such as "a little" and "approximately", can be given as $DIL(A)$ operator defined as follows:

$$DIL\,(A) \equiv A^{0.5} = \int_X \sqrt{A(x)}/x.$$

In addition, different fuzzy operators are used in fuzzy logic. For example, the meaning of the term "extremely A" can be defined as follows:

$$''\text{Extremely A}'' \equiv CON(CON(A)) \ = \int_X [A(x)]^4/x$$

In addition, the contrast intensification operator that reduces the fuzziness can be defined as follows:

$$''\text{Really A}'' \equiv \text{INT}\,(\text{A}) = \begin{cases} \displaystyle\int_X 2[A(x)]^2/x, & 0 \le A\,(x) \le 0.5, \\ \displaystyle\int_X [1 - 2[1 - A\,(x)]^2]/x, & 0.5 < A\,(x) \le 1. \end{cases}$$

The effects some of the operators given above on the basic "Young" membership function are shown in Figure 2 as an example.

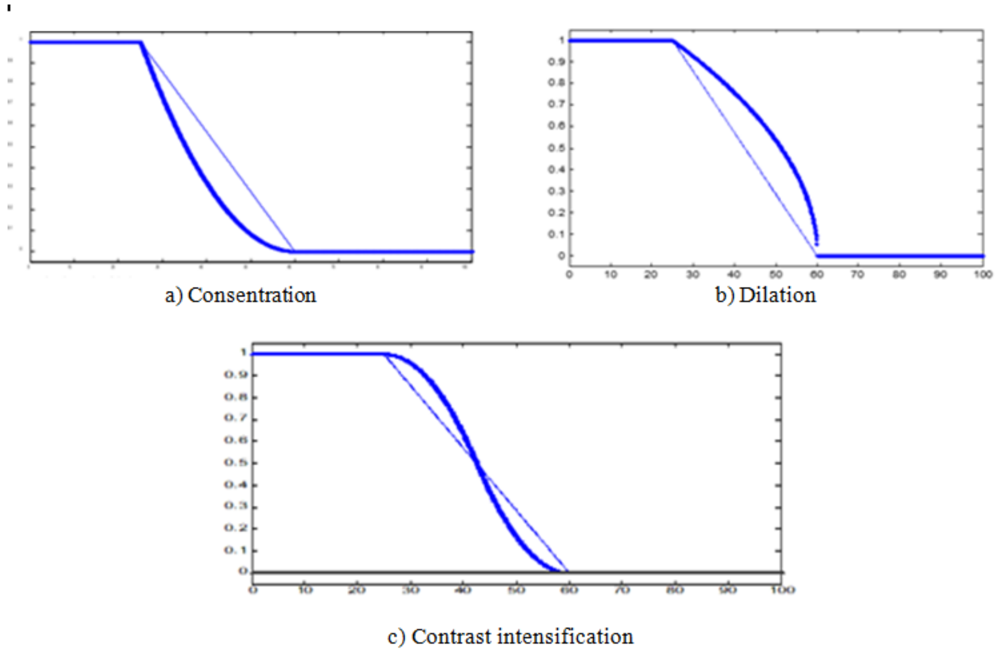


Figure 2: Effects of the a) Consentration, b) Dilation and c) Contrast intensification operators on the membership function

In the following examples, the basic linguistic values of "Young" and "Old" of the linguistic variable "Age“ are given in Figure 3 and examples of composite linguistic values derived on them through logical expressions are shown in Figure 4.

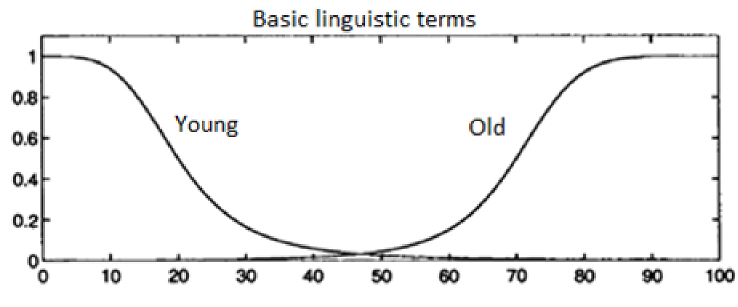


Figure 3: Graphs of the basic linguistic terms "young" and "old"

2.2 Fuzzy inference

Fuzzy inference generally refers to the rule-based inference mechanism as follows:

Rule: ***if*** $\boldsymbol{x = A}$ ***then*** $\boldsymbol{y = B}$
Fact: $\mathbf{x} = \widetilde{\mathbf{A}} \approx \mathbf{A}$
———————————————————————

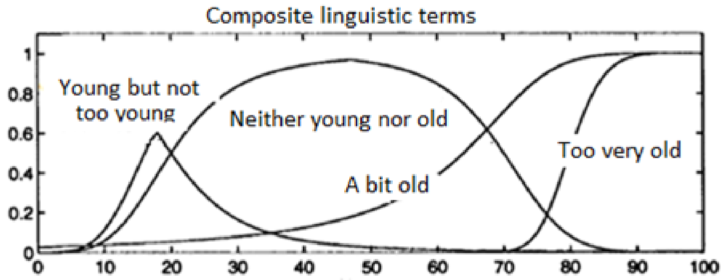


Figure 4: Composite linguistic values derived from the basic linguistic terms "young" and "old"

Result: $\mathbf{y}=\widetilde{\mathbf{B}} \approx \mathbf{B}$

Here the sign $\approx$ means "approximately equality" and the degree of "approximately equal" of the two fuzzy values A_1 and A_2 is generally calculated as follows in fuzzy logic:

$$\mu\left(A_1 \approx A_2\right) = \max_x \min\{A_1\left(x\right), A_2\left(x\right)\}$$

Considering this, the following formula can be used to calculate the result value of $\tilde{B}\left(y\right)$ according to the rule:

$$\tilde{B}\left(y\right) = \min\left\{\bigvee_x \left(\tilde{A}\left(x\right) \wedge A\left(x\right)\right), B\left(y\right)\right\} = \min\left\{w, B\left(y\right)\right\}$$

Here

$$w = \bigvee_x \left(\tilde{A}\left(x\right) \wedge A\left(x\right)\right),$$

determines the degree of provision of the antecedent part of the rule, i.e. the activation degree of the rule.

The design of the fuzzy reasoning mechanism is realized through the Fuzzy Inference System (FIS). Commonly used FIS designs are Mamdani and Sugeno type systems (Jang et al., 1997). The general design of the fuzzy inference system is given in Figure 5.

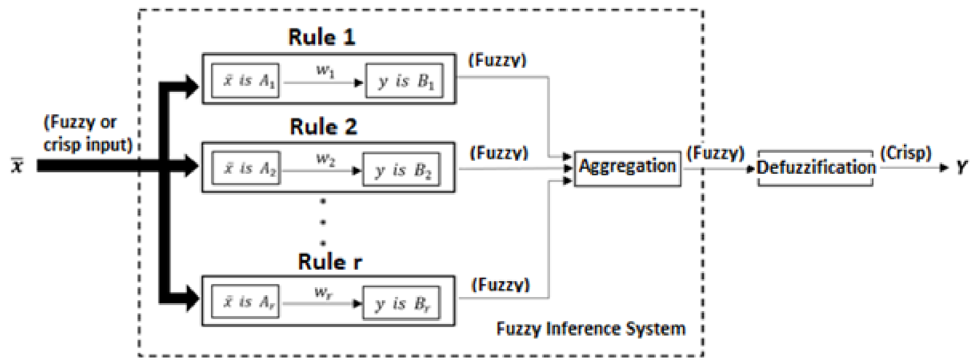


Figure 5: General scheme of the Fuzzy Inference System (FIS)

An exemplary Mamdani type FIS working diagram with precise and fuzzy inputs is given in Figure 6.

Figure 7 shows an example of a first-order Sugeno FIS with two rules and two inputs (one precise and another fuzzy).

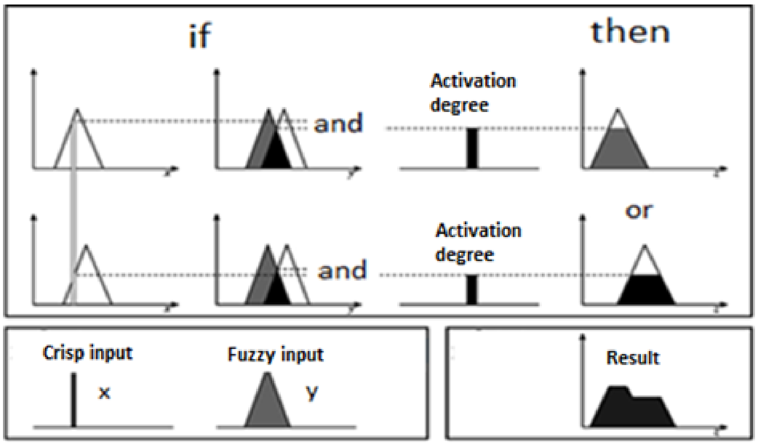


Figure 6: Example of a Mamdani type FIS with two rules and two precise and fuzzy inputs

2.3 Design and working principle of the FIS

For FIS design, the following two basic definitions are generally required (Jang et al., 1997):
- Defining the rules
- Defining the membership functions or result functions of fuzzy terms used in the rules.

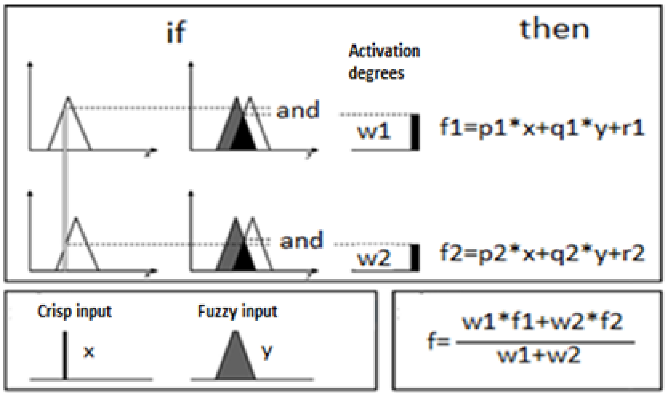


Figure 7: Example of a Sugeno type FIS with two rules and two inputs (one precise and another fuzzy

The FIS working principle may be carried out in the following steps:

Step 1. Calculate the firing (activation) degree of each rule.

Step 2. The FIS result may be calculated in two different ways: Mamdani and Sugeno type FIS:

Step 3. In case of Mamdani type FIS:

Step 3.1. The fuzzy result of each rule is truncated by rule's degree of activation, and the FIS overall result is found to be the aggregation of the results of the rules.

Step 3.2. If necessary, defuzzification is also applied to the overall fuzzy result.

In case of Sugeno type FIS:

Step 3.1. The overall FIS result is calculated as the weighted average of the result function values of the rules to the appropriate degree of activation.

3 Verbal stated mathematical problem and a solution approach

In real life situations, problems are not generally defined in line with the requirements of classical mathematics, using numerical values and precise logical inferences. Usually, data are expressed in verbal terms, and inferences are expressed in ordinary sentences rather than by precise mathematical formulations (Zadeh, 1965, 1975). We will call these defined problems as Verbal Stated Mathematical Problems.

Definition: *Verbal Stated Mathematical Problem (VSMP) is a text-defined problem that requires calculations or logical inferences based on linguistic data or concepts.*

The following is an example of a VSMP.

VSMP example: According to the expert opinion, a young person should consume approximately 20 g per day and an old person should consume about 50 g per day from certain nutrition. In this case, calculate how many grams a day should be consumed by someone who is not too old.
To resolve a VSMP, the following steps are often required:

Step 1. The verbal concepts mentioned in the text and used in the solution of the problem should be determined. For example, “young person”, “old person”, “approximately 20 grams”, “approximately 50 grams”.

Step 2. Linguistic input and output variables corresponding to verbal concepts should be created. For example, “age” is a verbal input variable and “nutrient amount” is a linguistic output variable.

Step 3. The verbal inferences used in the verbal description of the problem should be determined. For example, “a young person should consume approximately 20 g”, “an elderly person should consume approximately 50 g”.

Step 4. The Fuzzy Inference System corresponding to linguistic inference should be determined. For example, Mamdani type FIS, Sugeno type FIS etc. (Jang et al., 1997).

Step 5. Based on verbal inferences in the definition of the problem, the rules of the Fuzzy Inference System (FIS) should be determined. For example, “Rule 1: if x is “Young” then y is ” approximately 20 g”, “ Rule 2: if x is ”Old” then y is ” approximately 50 g”.

Step 6. The fuzzy result through FIS should be calculated.

Step 7. If necessary, the fuzzy result should be defuzzified by a specific defuzzification method. For example, Centroid of the Area (COA), Mean of the Maxima (MOM), Weighted Averaging Based on Levels (WABL) et al. (Jang et al., 1997; Nasibova & Nasibov, 2010; Nasiboglu & Erten, 2019).

In the following section, an example of the VSMP and its solution steps are given.

4 An example of the verbal stated mathematical problem and its solution steps

This section will describe the steps of solving a VSMP example as follows.

VSMP example: According to the expert’s opinion, suppose that from a particular type of nutrition, a **young person** should consume **20 g per day**, and an **old person** should consume **50 g per day**. In this case:

1. How many grams per day needs a **45-year-old person** to consume?
2. How many grams per day should **someone not too old** consume?

In the VSMP text above, the terms to be used in the solution process are indicated in bold. From the nature of the problem, it is seen that Sugeno type FIS should be used for the solution. The steps to be followed for the solution are given in the next subsections.

4.1 Design of a Sugeno type FIS

As it appears from the problem, Sugeno type FIS should be used to calculate the results. Let's first determine fuzzy rules.
Step 1. Determination of the rules:

1. **Rule 1: if** 'Young' **then** result 1 = 20 g
2. **Rule 2: if** 'Old' **then** result 2 = 50 g

Step 2. Determination of the terms:

$$\mu_{Young}(x) = \begin{cases} 1, & x < 25, \\ \frac{60-x}{35}, & x \in [25,\ 60], \\ 0, & x > 60. \end{cases}$$

$$\mu_{Old}(x) = \begin{cases} 0, & x < 40, \\ \frac{x-40}{35}, & x \in [40,\ 75], \\ 1, & x > 75. \end{cases}$$

Let be A="Not Very Old" = "Not (Very Old)". Then we get:

$$\mu_A(x) = 1 - CON(\mu_{Old}(x)) = 1 - \mu_{Old}(x)^2 = \begin{cases} 1, & x < 40, \\ 1 - \left[\frac{x-40}{35}\right]^2, & x \in [40, 75], \\ 0, & x > 75. \end{cases}$$

With respect to the definitions of the example problem, the graphics of "Young", "Old" and "Not Very Old" terms are given in Figure 8.

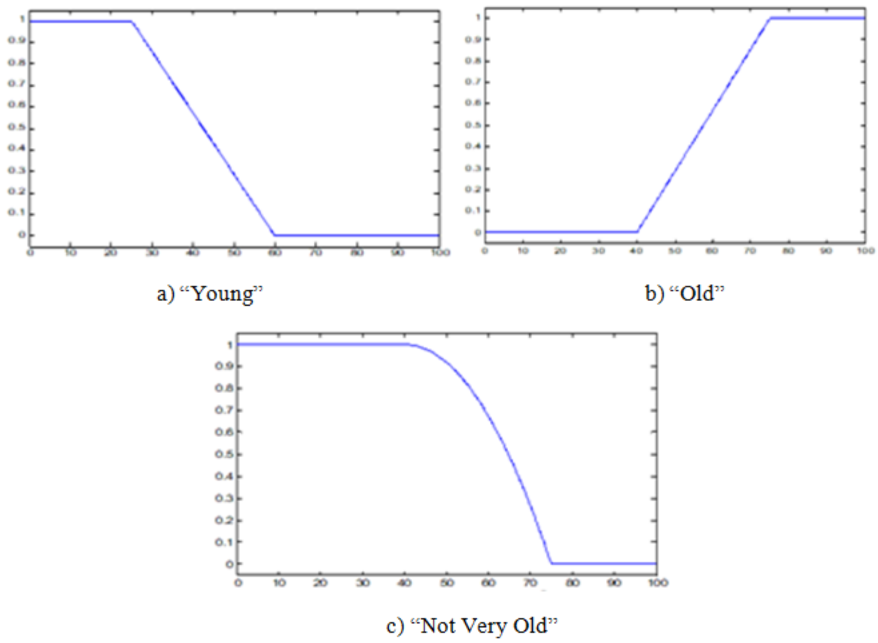


Figure 8: Graphics of the terms a) "Young", b) "Old" and c) "Not Very old"

Step 3. a) For a 45-year-old, the exact input FIS will be used to calculate the result.

1. *If rule 1 is applied:* To what degree is a 45-year-old one young?

$$\mu_{Young}(45) = \frac{60-45}{35} = 0.43$$

2. So according to Rule 1 the result should be 20 grams with 0.43 degrees.
3. *If rule 2 is applied:* How old is a 45-year-old person?

$$\mu_{Old}(45) = \frac{45-40}{35} = 0.14$$

4. So according to Rule 2, the result should be 50 grams with 0.14 degrees.
5. *Aggregated result* will be calculated as the weighted average of the results of all rules:

$$FIS(45) = \frac{0.43 * 20 + 0.14 * 50}{0.43 + 0.14} = 27.37g$$

Step 3. b) For a "not very old" person, the fuzzy input FIS will be used to calculate the result. Thus, for someone A = "not very old", the FIS result will be calculated as follows:

1. *If rule 1 is applied:* To what degree is a "not very old" one young?

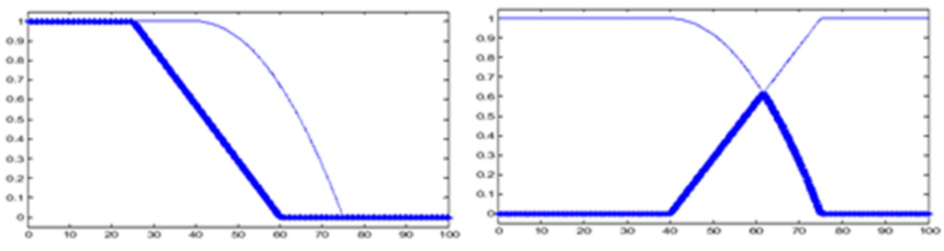
$$\max_x\{\min(\mu_{Young}(x), \mu_A(x))\} = 1.0$$

2. So according to Rule 1, the result should be 20 grams with 1.0 degree.
3. *Let's follow Rule 2.* To what degree is old a "not very old" someone?

$$\max_x\{\min(\mu_{Old}(x), \mu_A(x))\} = 0.62$$

4. So according to Rule 2, the result should be 50 grams with 0.62 degrees.
5. *Aggregated result* will be calculated as the weighted average of the results of all rules:

$$FIS(\text{"not very old"}) = \frac{1.0 * 20 + 0.62 * 50}{1.0 + 0.62} = 31.48g$$



a) Activation degree of the Rule 1.   b) Activation degree of the Rule 2.

Figure 9: Graphical representation of the calculations of the activation degree of a) Rule 1 and b) Rule 2 for "not very old" fuzzy input

The calculation of the firing degrees of the rules for the "not very old" fuzzy input is given in Figure 9.

5 Conclusion

In this article, the concept of Verbal Stated Mathematical Problem (VSMP) was first defined and a methodological approach based on fuzzy sets and fuzzy logic techniques was proposed for the solution of such problems. In this methodological approach, the solution is examined primarily through Sugeno type FIS. This type of FIS is used, because the input parts of the handled problem example are fuzzy sets such as "45-year-old person" and "someone not too old", and the output part is precise numbers such as "20g" and "50g". If fuzzy values such as "approximately 20 g", "approximately 50 g" were used in the output parts instead of exact numbers, it would be more appropriate to use Mamdani type FIS. In this case, the FIS output would have to additionally be subjected to the defuzzification operation.

Let denote that other FIS types can be used depending on the stated problem types. In our future studies, it is considered to address the problems that can be solved through various other types of Approximate Reasoning techniques and to develop appropriate solution methodologies. In addition, studies on the ability to extract and analyze VSMP's in the text can be continued by using various Text Mining and Deep Learning technologies such as Text Summarization, Named Entity Recognition etc.

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